

## 5.7 Rational Exponents

- write expressions with rational exponents in radical form and vice versa
- simplify

For all

$$b \in \mathbb{R}$$

$$n \in \mathbb{Z}$$

$$\sqrt[n]{b} = b^{\frac{1}{n}}$$

Exponential Form

Radical Form

$8^{\frac{1}{3}}$	$\sqrt[3]{8} = 2$
$64^{\frac{1}{2}}$	$\sqrt{64} = 8$
$16^{\frac{1}{4}}$	$\sqrt[4]{16} = 2$
$x^{\frac{1}{3}}$	$\sqrt[3]{x}$
$x^{\frac{3}{4}}$	$\sqrt[4]{x^3}$

$$b^{\frac{m}{n}} = \sqrt[n]{b^m}$$

For all  $b \in \mathbb{R}$  ( $b \neq 0$ ) and  $m, n \in \mathbb{Z}$  ( $n > 1$ )

Simplified

- no negative exponents
- no fractional exponents in denominator
- not a complex fraction
- index is as low as it can be

Simplify.

$$\sqrt[4]{36x^2}$$

Handwritten work for  $\sqrt[4]{36x^2}$ :

$(6^2 x^2)^{\frac{1}{4}}$   
 $6^{\frac{2}{4}} x^{\frac{2}{4}}$   
 $6^{\frac{1}{2}} x^{\frac{1}{2}}$   
 $(6x)^{\frac{1}{2}}$

Final simplified form:  $\sqrt{6x}$

Simplify.

$$\sqrt[8]{16}$$

Handwritten work for  $\sqrt[8]{16}$ :

$(4^2)^{\frac{1}{8}}$   
 $4^{\frac{2}{8}}$   
 $\sqrt[4]{4} = \sqrt[4]{2^2}$   
 $\sqrt{2} \leftarrow 2^{\frac{1}{2}}$

Handwritten work for  $\sqrt[8]{16}$ :

$\sqrt[8]{16} = (2^4)^{\frac{1}{8}}$   
 $2^{\frac{4}{8}}$   
 $2^{\frac{1}{2}}$   
 $\sqrt{2}$

Other handwritten work:

$\sqrt[4]{\sqrt{16}}$   
 $\sqrt[4]{4}$   
 $\sqrt[4]{4} = \sqrt{2}$

Simplify.

$$\sqrt[15]{32} = (2^5)^{\frac{1}{15}} = 2^{\frac{5}{15}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$$

32  
16 2  
4 4  
2 2 2

Add exponents

$$x^{\frac{2}{3}} \cdot x^{\frac{1}{2}} = x^{\frac{4}{6} + \frac{3}{6}} = x^{\frac{7}{6}} = \sqrt[6]{x^7} = \boxed{x\sqrt[6]{x}}$$

$$\sqrt[12]{9x^6}$$

~~$\sqrt[12]{3^2 \cdot x^6}$~~

$(3^2 x^6)^{\frac{1}{12}}$   
 $(3 x^3)^{\frac{2}{12}}$   
 $(3 x^3)^{\frac{1}{6}}$   
 $\sqrt[6]{3x^3}$

$$\frac{\sqrt[8]{16}}{\sqrt[6]{2}} = \frac{2^{\frac{4}{8}}}{2^{\frac{1}{6}}} = \frac{2^{\frac{1}{2}}}{2^{\frac{1}{6}}} = 2^{\frac{1}{2} - \frac{1}{6}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$\frac{3}{y^{\frac{1}{2}}} \quad \sqrt{y}$$

HW  
p261  
21-61odd